Math 270: Differential Equations Day 1 – What Are Differential Equations?

<u>What is the difference between an algebra equation and a differential equation?</u>

1. In an algebra equation you're solving for an unknown number

In a differential equation you're solving for an unknown function

What is the difference between an algebra equation and a differential equation?

2. In a differential equation there will be at least 1 appearance of the 1^{st} or higher order derivative of the unknown function

Notation

What is the difference between an algebra equation and a differential equation?

2. In a differential equation there will be at least 1 appearance of the 1^{st} or higher order derivative of the unknown function

Definitions:

- The highest derivative appearing on the unknown function in the differential equation is called the order of the differential equation

What is the difference between an algebra equation and a differential equation?

2. In a differential equation there will be at least 1 appearance of the 1st or higher order derivative of the unknown function

Definitions:

- The differential equation is linear if it can be written in the following form

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = F(x)$$

What is the difference between an algebra equation and a differential equation?

2. In a differential equation there will be at least 1 appearance of the 1^{st} or higher order derivative of the unknown function

Definitions:

- A differential equation is an ordinary differential equation (ODE) if the unknown function is a function of a single variable (calc 1 function)

What is the difference between an algebra equation and a differential equation?

2. In a differential equation there will be at least 1 appearance of the 1^{st} or higher order derivative of the unknown function

Definitions:

- A differential equation is an partial differential equation (PDE) if the unknown function is a function of more than 1 variable (and derivatives are partial derivatives)

What is the difference between an algebra equation and a differential equation?

3. When checking a solution in an algebra equation…

Is $x = 2$ a solution to $\frac{x^3-2}{x+1} = x$?

Both sides are numbers

When checking a solution in a differential equation…

Is $y = x^2$ a solution to $xy'' + xy' = 2(y + x)$?

Both sides are functions/expressions

What is the difference between an algebra equation and a differential equation?

4.

Many Types of
Algebra Equations

$2x + 5 = 9$ (linear)

$x^2 + 5x - 1 = 0$ (quadratic)

$\sqrt{x-4} + \sqrt{x} = \sqrt{x+1}$ (radical)

$\frac{x}{x+1} + \frac{x-3}{x+2} = x$ (rational)

Many Types of
Differential Equations

$y' + 2xy = 3$ (first order linear)

$y' = 2xsiny$ (seperable)

What is the difference between an algebra equation and a differential equation?

5. Let's Guess Solutions to Some DEs!

a) $y' = 2x$

b) $y' = \cos x$

c) $y' = y$

d) $y' = 3y$

e) $y'' = -y$

f) $y'' = -4y$

What is the difference between an algebra equation and a differential equation?

6. Note:
   - When solving a DE, you are looking for ALL of the solutions.
   - An Initial Value Problem IVP is a DE along with a point on the graph of the function. Of all the solutions to the DE, the point singles out one of them, so an IVP typically has only 1 solution

Illustrate with IVP: $y' = y\,, \quad y(0) = 4$

What is the difference between an algebra equation and a differential equation?

7. Why do we care about differential equations?